

1. The force, \vec{F} , illustrated in the figure shown below has a magnitude of 100 N.
- Express the force, \vec{F} , in Cartesian vector notation.
 - Find the magnitude of the component of vector \vec{F} that lies in the direction of the displacement vector, \vec{r} .
 - Write out expressions for the two vectors, $\vec{F}_{||}$ and \vec{F}_{\perp} . $\vec{F}_{||}$ is the vector component of \vec{F} that lies in the direction of \vec{r} . \vec{F}_{\perp} is the second vector component of \vec{F} that is perpendicular to $\vec{F}_{||}$.

Solution:

$$(a) \quad \vec{F} = -100 \cos 30^\circ \sin 45^\circ \hat{i} + 100 \cos 30^\circ \cos 45^\circ \hat{j} + 100 \sin 30^\circ \hat{k}$$

$$\vec{F} = \{-61.24 \hat{i} + 61.24 \hat{j} + 50 \hat{k}\} \text{ N}$$

$$(b) \quad F_{||} = \hat{u}_r \cdot \vec{F} ; \quad \hat{u}_r = \frac{\vec{r}}{r}$$

$$\vec{r} = \{(-2-0)\hat{i} + (0-6)\hat{j} + 3\hat{k}\}$$

$$|\vec{r}| = \sqrt{(-2)^2 + (-6)^2 + (3)^2} = 7.0 \text{ m}$$

$$\hat{u}_r = -0.2857 \hat{i} - 0.8571 \hat{j} + 0.4286 \hat{k}$$

$$F_{||} = \hat{u}_r \cdot \vec{F} \text{ (scalar, dot prod.)}$$

$$F_{||} = -0.2857(-61.24) + (-0.8571)(61.24) + 0.4286(50)$$

$$F_{||} = -13.56 \text{ N}$$

$$(c) \quad \vec{F}_{||} = \hat{u}_r (\hat{u}_r \cdot \vec{F}) = -13.56(-0.2857 \hat{i} - 0.8571 \hat{j} + 0.4286 \hat{k})$$

$$\vec{F}_{||} = \{3.874 \hat{i} + 11.622 \hat{j} - 5.812 \hat{k}\} \text{ N}$$

$$\vec{F}_{\perp} + \vec{F}_{||} = \vec{F} ; \quad \vec{F}_{\perp} = \vec{F} - \vec{F}_{||} \text{ expression}$$

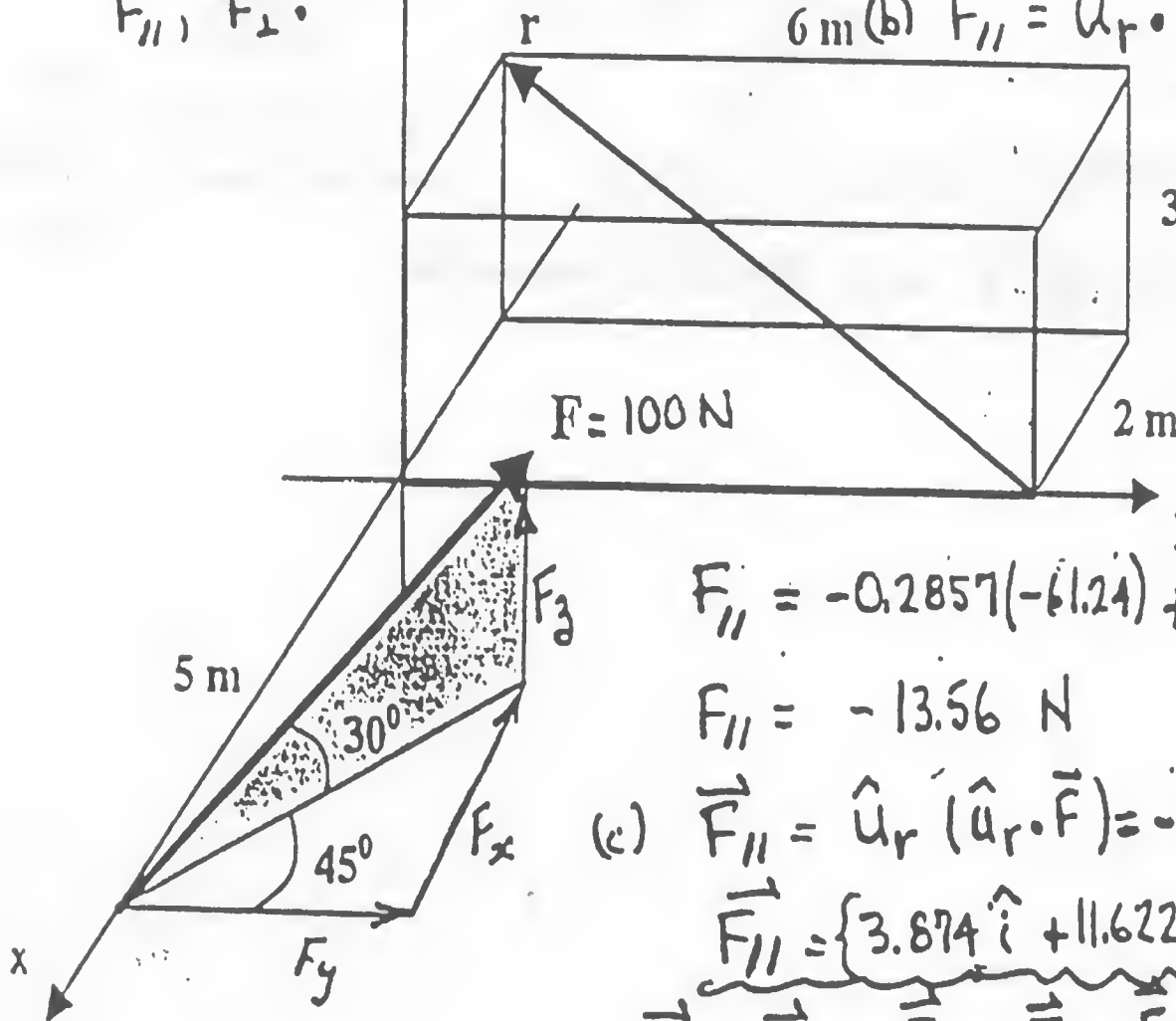
$$\vec{F}_{\perp} = \{-61.24 \hat{i} + 61.24 \hat{j} + 50 \hat{k}\} - \{3.874 \hat{i} + 11.622 \hat{j} - 5.812 \hat{k}\}$$

$$\vec{F}_{\perp} = \{-65.1 \hat{i} + 49.6 \hat{j} + 55.2 \hat{k}\} \text{ N}$$

Given: Stated Problem

Find: \vec{F} , $F_{||}$,

$\vec{F}_{||}$, \vec{F}_{\perp} .



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- 2a) A load Q is applied to the pulley C , which can roll on the cable ACB . The pulley is held in the position shown by a second cable CAD , which passes over the pulley A and supports a load P . Knowing that $P = 750$ N, determine the tension in cable ACB and the magnitude of load Q .

Given: Stated Problem & Figure

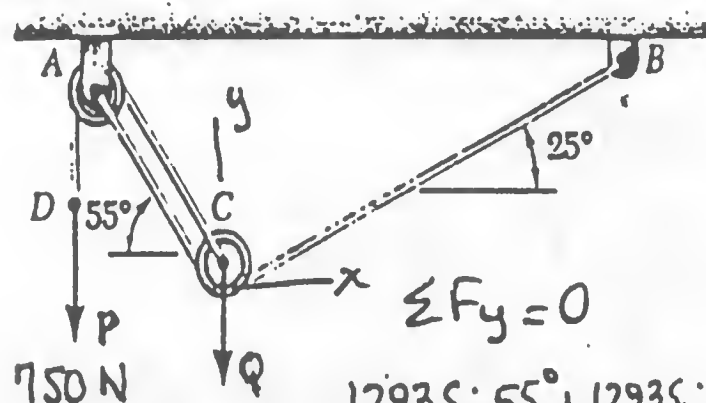
Find: F_{ACB} cable tension & Q .

Soln: Equilibrium Problem (2D).

$$\sum F_x = 0 ; F_{CA} = F_{CB}$$

FBD Pulley C :

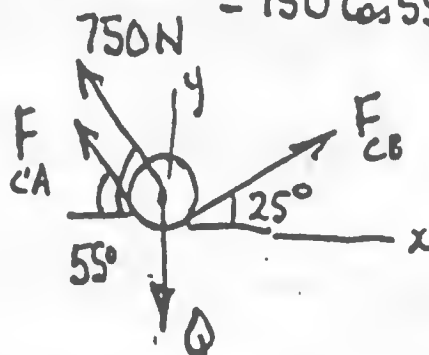
$$-750 \cos 55^\circ - F \cos 55^\circ + F \cos 25^\circ = 0$$



$$\sum F_y = 0$$

$$1293 \sin 55^\circ + 1293 \sin 25^\circ + 750 \sin 55^\circ - Q = 0$$

$$Q = \underline{2220 \text{ lb}} \quad \text{Load directed down as shown}$$



$$-430.2 - 0.5736F + 0.9063F = 0$$

$$F = \frac{430.2}{0.3327}$$

$$F = \underline{1293 \text{ N}}$$

cable
tension
ACB

2b) Three cables are used to tether a balloon as shown. Determine the vertical force \vec{P} exerted by the balloon at A knowing that the tension in cable AB is 259 N.

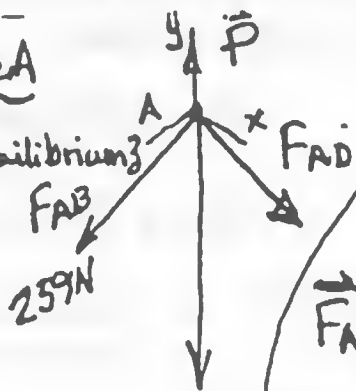
Given: Problem shown.

Find: \vec{P}_y .

Solution: \rightarrow

FBD @ A

Pt. A is in equilibrium?



3D Equilibrium Problem.

Write each force in

Vector format: Place in eqn.

Then $\sum F_x = 0; \sum F_y = 0; \sum F_z = 0$

$$\vec{F}_{AB} = 259 \hat{u}_{AB} = 259 \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} \text{ lb}$$

$$\vec{r}_{AB} = (-4.20 - 0)\hat{i} + (0 - 5.6)\hat{j} + 0\hat{k} \text{ m}$$

$$|\vec{r}_{AB}| = 7.0 \text{ m} \leftarrow \sqrt{(-4.2)^2 + (-5.6)^2} \text{ etc.}$$

$$\vec{F}_{AB} = \left\{ 259 \left[\left(\frac{-4.2}{7.0} \right) \hat{i} - \left(\frac{5.6}{7.0} \right) \hat{j} \right] \right\} \text{ N}$$

$$\vec{F}_{AB} = \{-155.4 \hat{i} - 207.2 \hat{j} + 0 \hat{k}\} \text{ N}$$

$$\vec{F}_{AD} = F_{AD} \hat{u}_{AD} = F_{AD} \frac{(0\hat{i} - 5.6\hat{j} - 3.3\hat{k})}{6.5} \text{ N}$$

$$\vec{F}_{AC} = F_{AC} \hat{u}_{AC} = F_{AC} \frac{(2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k})}{7.40} \text{ N}$$

$$\vec{P} = P \hat{j} \quad \leftarrow \sqrt{(2.4)^2 + (-5.6)^2 + (4.2)^2} = 7.40$$

$\sum F_x = 0$ collect \hat{i} 's & solve: (\hat{i} 's = x's)

$$-155.4 \hat{i} + F_{AC} 0.324 \hat{i} = 0; \underline{F_{AC} = 479.2 \text{ N}}$$

$\sum F_z = 0$ collect \hat{k} 's & solve:

$$479.2 \frac{(4.2)}{7.4} \hat{k} - F_{AD} \frac{(3.3)}{6.5} \hat{k} = 0; \underline{F_{AD} = 535.7 \text{ N}}$$

$\sum F_y$ for \vec{P} , collecting \hat{j} 's:

$$P \hat{j} - 207.2 \hat{j} - 535.7 \left(\frac{5.6}{6.5} \right) \hat{j} - 479.2 \left(\frac{5.6}{7.4} \right) \hat{j} = 0$$

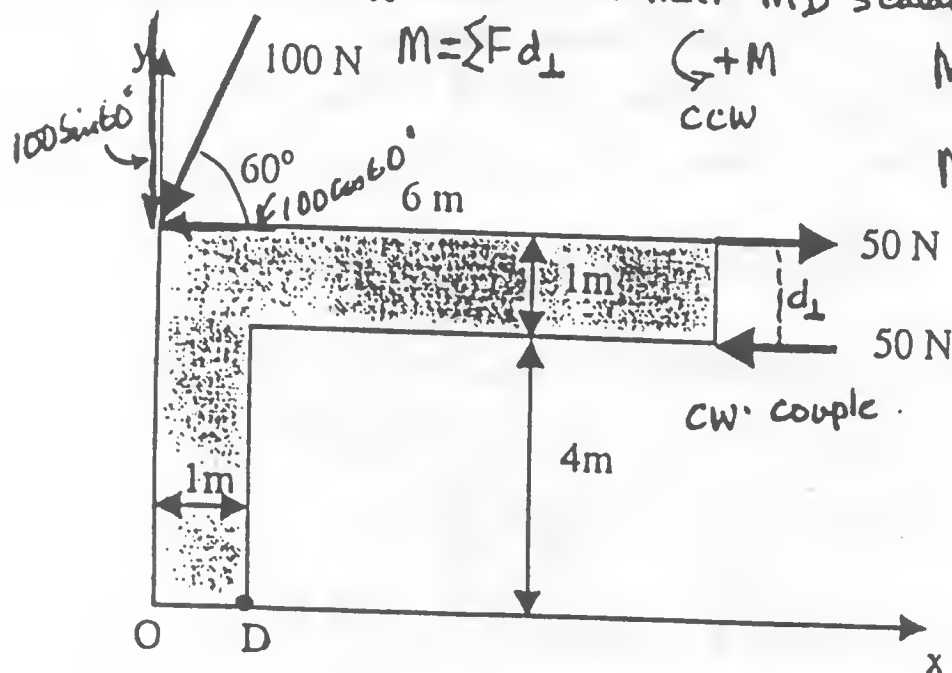
$$\underline{\vec{P} = 1031.3 \hat{j} \text{ N}} \text{ or } \underline{1031 \hat{j} \text{ N}}$$

3a) The two-dimensional object experiences the three forces shown. Using scalar methods, calculate the moment at point D, located 1m from O along the x-axis.

Given: Problem shown.

Find: M_D scalar method.

2D-Moment Summation Problem. Sol'n:



$$M_D = 100 \cos 60^\circ (5) + 100 \sin 60^\circ (1) - 50(1) \text{ Nm}$$

$$M_D = 286.6 \text{ Nm} \text{ CCW}$$

$$\text{or } M_D = 287 \text{ Nm} \text{ CCW}$$

3b) Calculate the moment vector \vec{M}_{mn} generated by the 200 lb force about axis m-n using Cartesian vectors. Point n lies on the y-z plane, 4 ft right and 2 ft down from O.

Given: Problem shown.

Find: \vec{M}_{mn}

Sol'n:

$$\vec{M}_{mn} = \hat{u}_{mn} (\hat{u}_{mn} \cdot [\vec{r}_{mo} \times \vec{F}_3]) ; \hat{u}_{mn} = \frac{\vec{r}_{mn}}{|\vec{r}_{mn}|}$$

$$\hat{u}_{mn} = \frac{(0-3)\hat{i} + (4-0)\hat{j} + (-2-0)\hat{k}}{\sqrt{(-3)^2 + (4)^2 + (-2)^2}} = \frac{-3\hat{i} + 4\hat{j} - 2\hat{k}}{5.3852}$$

$$\hat{u}_{mn} = \{-0.5571\hat{i} + 0.7428\hat{j} - 0.3714\hat{k}\}$$

$$\vec{r}_{mo} = (-3\hat{i} + 0\hat{j} + 0\hat{k}) \text{ ft}$$

$$\vec{F}_3 = (200\hat{k}) \text{ lb}$$

$$M_{mn} = \hat{u}_{mn} \cdot \vec{r}_{mo} \times \vec{F}_3$$

$$M_{mn} = \begin{vmatrix} -0.5571 + 0.7428 & -0.3714 \\ -3.00 & 0 & 0 \\ 0 & 0 & 200 \end{vmatrix} \text{ ft}\cdot\text{lb}$$

$$M_{mn} = -0.7428(-3.00)200 = 445.7 \text{ ft}\cdot\text{lb}$$

$$\text{From } \vec{M}_{mn} = (-0.5571\hat{i} + 0.7428\hat{j} - 0.3714\hat{k}) 445.7 \text{ ft}\cdot\text{lb}$$

$$\therefore \vec{M}_{mn} = \{-248.3\hat{i} + 331.1\hat{j} - 165.5\hat{k}\} \text{ ft}\cdot\text{lb}$$

$$\text{or } \vec{M}_{mn} = \{-248\hat{i} + 333\hat{j} - 165.5\hat{k}\} \text{ ft}\cdot\text{lb} \text{ (rounded)}$$

